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## THE INFLUENCE OF TIP MASS OFFSET ON THE STABILITY OF BECK'S COLUMN

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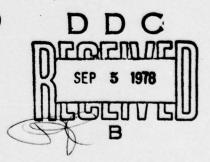
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#### TABLE OF CONTENTS

|   | Page |
|---|------|
| INTRODUCTION  | 1    |
| STATEMENT OF PROBLEM  | 2    |
| THE EXACT SOLUTION  | 4    |
| NUMERICAL RESULTS   | 6    |
| REFERENCES  | 12   |
| LIST OF ILLUSTRATIONS   |      |
| Figure  |      |
| <ol> <li>A cantilever beam carrying a tip mass and subjected<br/>to a tangenital follower force.</li> </ol> | 3    |
| 2. Location of the centroid of the tip mass   | 7    |
| 3. Variation of $Q_{ca}/\pi^{2}$ versus $\xi$ for four values of p.   | 11   |

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### THE INFLUENCE OF TIP MASS OFFSET ON THE STABILITY OF BECK'S COLUMN

In this paper, the stability of a slender cantilever carrying a tip mass at its free end and subjected there to a follower force is investigated. The centroid of the tip mass is offset from the free end of the beam and is located along its extended axis. The associated boundary value problem is solved and the exact frequency equation is derived. The frequency equation is solved numerically for the case in which both the beam and the tip mass have circular cross sections. The numerical computations indicate that the system loses stability only through flutter. The variation of the values of the critical flutter load  $Q_{\rm cr}$  with the tip mass offset parameter  $\xi$  is shown graphically for four values of the tip mass density to beam density ratio p. These calculations reveal that, at sufficiently small values of  $\xi$ ,  $Q_{\rm cr}$  decreases sharply for increasing values of p. For values of  $\xi$  sufficiently large, however, the situation is reversed as the value of  $Q_{\rm cr}$  increases with increasing p.

#### 1. INTRODUCTION

An elastic cantilever of length  $\ell$  and density  $\rho$  that is subjected to a compressive follower force of magnitude P applied at its free end is known as Beck's column [1]. Pfluger [2] has investigated the influence of the transverse inertia of a tip mass on the value of the critical flutter load of Beck's column, and Walter and Levinson [3] and Anderson [4] have considered closely related problems with the inclusion of the influence of the rotatory inertia of the tip mass. If the rotatory inertia of the tip mass is negligible, its transverse inertia generally tends to reduce the value of the critical flutter load. However, if the mass of the column is

sufficiently small relative to that of the body attached at its tip, the value of the critical flutter load can be slightly greater than Beck's value of  $P_{cr} = 20.05 \text{ EI/}\ell^2$ .

In all the studies mentioned above, it was assumed that the mass of the attached body was concentrated at the free end of the column. As a further step in studying the influence of a tip mass on the stability of Beck's column, one may consider the system in which the centroid of the tip mass is offset from the point of attachment along the extended axis of the column. Only recently the natural frequencies of free vibration of a uniform unloaded cantilever carrying such a tip mass have been reported by Bhat and Wagner [5], Bhat and Kulkarni [6], and Flax [7].

#### 2. STATEMENT OF THE PROBLEM

The differential equation of motion and the boundary conditions for the system consisting of Beck's column carrying at its free end a tip mass whose centroid is offset from the point of attachment (see Figure 1) are

EI 
$$\frac{\partial^4 w}{\partial x_1^4} + P \frac{\partial^2 w}{\partial x_1^2} + \rho A \frac{\partial^2 w}{\partial t^2} = 0$$
,  $0 < x_1 < \ell$ , (1)

$$w(0,t) = \frac{\partial w}{\partial x_1}(0,t) = 0$$
, (2)

$$EI \frac{\partial^2 w}{\partial x_1^2}(\ell,t) = -(J_0 + mc^2) \frac{\partial^3 w}{\partial x_1 \partial t^2}(\ell,t) - mc \frac{\partial^2 w}{\partial t^2}(\ell,t) , \qquad (3)$$

EI 
$$\frac{\partial^3 w}{\partial x_1^3}(\ell,t) = m \frac{\partial^2 w}{\partial t^2}(\ell,t) + mc \frac{\partial^3 w}{\partial x_1 \partial t^2}(\ell,t)$$
, (4)

where  $w(x_1,t)$  denotes the transverse deflection of the column, EI its flexural rigidity, P is the magnitude of the applied force,  $\rho$  the density

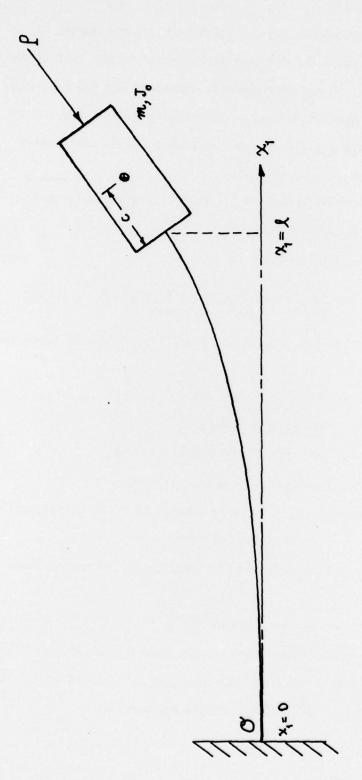


Figure 1. A cantilever beam carrying a tip mass and subjected to a tangential follower force.

of the beam, A its cross-sectional area,  $\ell$  its length,  $J_0$  the moment of inertia of the tip mass, and c the distance from the end of the beam to the centroid of the tip mass. If one sets c=0 in equations (3) and (4), then the given boundary value problem reduces to the problem that is the adjoint of the one solved in reference [3], whereas, if both c and  $J_0$  are equated to zero, Pfluger's problem [2] is obtained.

It is expedient to express equations (1) to (4) in dimensionless form by making the changes of independent variables  $x_1 = \ell x$ , 0 < x < 1,  $t = g\tau$ , and by introducing the following parameters:

$$g^2 = \frac{\rho A \ell^4}{EI}$$
,  $Q = \frac{P \ell^2}{EI}$ ,  $\gamma = \frac{J_0}{\rho A \ell^3}$ ,  $\mu = \frac{m}{\rho A \ell}$ ,  $\alpha = \frac{c}{\ell}$ . (5)

As a consequence of these changes, equations (1) to (4) can now be expressed as follows:

$$w'''' + Qw'' + \ddot{w} = 0, \quad 0 < x < 1, \quad 0 < \tau,$$
 (6)

$$w(0,\tau) = w'(0,\tau) = 0,$$
 (7)

$$w''(1,\tau) + (\gamma + \mu\alpha^2)\ddot{w}'(1,\tau) + \mu\alpha\ddot{w}(1,\tau) = 0,$$
 (8)

$$w'''(1,\tau) - \mu w(1,\tau) - \mu \alpha w'(1,\tau) = 0,$$
 (9)

where primes and dots denote derivatives with respect to  $\boldsymbol{x}$  and  $\boldsymbol{\tau}$ , respectively.

#### 3. THE EXACT SOLUTION

The solution of the partial differential equation (6) is next assumed in the form

$$w(x,\tau) = y(x)e^{i\omega\tau}, \qquad (10)$$

where  $i = (-1)^{1/2}$  and a denotes the dimensionless natural frequency of vibration of the beam. Substitution of equation (10) into equations (6) to (9) yields the following non-self-adjoint eigenvalue problem:

$$y''''(x) + Qy''(x) - \omega^2 y(x) = 0, \quad 0 < x < 1,$$
 (11)

and

$$y(0) = y'(0) = 0$$
 (12)

$$y''(1) - \omega^{2}(\gamma + \mu\alpha^{2})y'(1) - \mu\alpha\omega^{2}y(1) = 0, \qquad (13)$$

$$y'''(1) + \mu \omega^2 y(1) + \mu \alpha \omega^2 y'(1) = 0$$
 (14)

It can be verified that the solution of equation (11) has the form

$$y(x) = A_1 \cos \lambda_1 x + A_2 \sin \lambda_1 x + A_3 \cosh \lambda_2 x + A_4 \sinh \lambda_2 x , \qquad (15)$$

where

$$\lambda_n = (1/\sqrt{2})[(Q^2 + 4\omega^2)^{1/2} - (-1)^n Q]^{1/2}, \quad n = 1, 2.$$
 (16)

Substitution of equation (15) into the boundary conditions in equations (12) to (14) yields the following system of homogeneous algebraic equations:

$$\sum_{k=1}^{4} a_{jk} A_{k} = 0, \quad j = 1,2,3,4$$
 (17)

where

$$\begin{aligned} \mathbf{a}_{11} &= 1\,, & \mathbf{a}_{12} &= 0\,, & \mathbf{a}_{13} &= 1\,, & \mathbf{a}_{14} &= 0\,, \\ \mathbf{a}_{21} &= 0\,, & \mathbf{a}_{22} &= \lambda_1\,, & \mathbf{a}_{23} &= 0\,, & \mathbf{a}_{24} &= \lambda_2\,, \\ \\ \mathbf{a}_{31} &= -(\lambda_1^2 + \omega^2\mu\alpha)\cos\lambda_1 + \lambda_1\omega^2(\gamma + \mu\alpha^2)\sin\lambda_1\,, \\ \mathbf{a}_{32} &= -(\lambda_1^2 + \omega^2\mu\alpha)\sin\lambda_1 - \lambda_1\omega^2(\gamma + \mu\alpha^2)\cos\lambda_1\,, \\ \\ \mathbf{a}_{33} &= (\lambda_2^2 - \omega^2\mu\alpha)\cosh\lambda_2 - \lambda_2\omega^2(\gamma + \mu\alpha^2)\sinh\lambda_2\,, \\ \\ \mathbf{a}_{34} &= (\lambda_2^2 - \omega^2\mu\alpha)\sinh\lambda_2 - \lambda_2\omega^2(\gamma + \mu\alpha^2)\cosh\lambda_2\,, \\ \\ \mathbf{a}_{41} &= \lambda_1(\lambda_1^2 - \omega^2\mu\alpha)\sinh\lambda_1 + \omega^2\mu\cos\lambda_1\,, \end{aligned}$$

$$\begin{aligned} \mathbf{a}_{42} &= -\lambda_1 (\lambda_1^2 - \omega^2 \mu \alpha) \cos \lambda_1 + \omega^2 \mu \sin \lambda_1, \\ \mathbf{a}_{43} &= \lambda_2 (\lambda_2^2 + \omega^2 \mu \alpha) \sinh \lambda_2 + \omega^2 \mu \cosh \lambda_2, \\ \mathbf{a}_{44} &= \lambda_2 (\lambda_2^2 + \omega^2 \mu \alpha) \cosh \lambda_2 + \omega^2 \mu \sinh \lambda_2. \end{aligned}$$

The system of equations (17) will have a non-trivial solution if and only if the determinant of the coefficient matrix vanishes, i.e.,

$$det(a_{ik}) = 0$$
.

Expansion of this determinant yields the frequency equation

$$Q^{2} + 2\omega^{2} + 2\mu\gamma\omega^{4} - \omega \left[Q(\omega^{2}\mu\gamma - 1) + \mu\alpha(Q^{2} + 4\omega^{2})\right]\sin\lambda_{1} \sinh\lambda_{2} - \omega\lambda_{2}(\lambda_{1}^{2} + \lambda_{2}^{2})\left[\mu + \lambda_{1}^{2}(\gamma + \mu\alpha^{2})\right]\sin\lambda_{1} \cosh\lambda_{2} + \omega\lambda_{1}(\lambda_{1}^{2} + \lambda_{2}^{2})\left[\mu - \lambda_{2}^{2}(\gamma + \mu\alpha^{2})\right]\cos\lambda_{1} \sinh\lambda_{2} + 2\omega^{2}(1 - \mu\gamma\omega^{2})\cos\lambda_{1} \cosh\lambda_{2} = 0.$$
 (18)

#### 4. NUMERICAL RESULTS

The objective of the numerical computations is to determine the dependence of the critical load parameter  $Q_{\rm cr}$  on the parameters  $\alpha$ ,  $\mu$ , and  $\gamma$ , which are inter-dependent. Hence, it is necessary to examine these quantities somewhat more closely. The moment of inertia parameter  $J_0$  is defined by

$$J_0 = \int_{\tau} (x_1^2 + z_2^2) dm, \qquad dm = \rho_0 d\tau, \tag{19}$$

where  $\tau$  denotes the volume of the tip mass and  $\rho_0$  its density. The  $z_1 z_2 z_3$  -coordinate axes have their origin at the centroid of the tip mass as shown in Figure 2. Since  $d\tau = dz_1 dz_2 dz_3$ , one has, under the assumption that the tip mass is a prismatic solid whose generators are parallel to the  $z_1$ -axis,

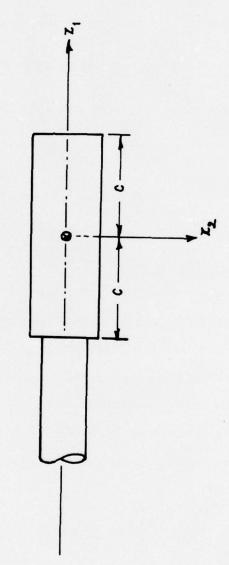


Figure 2. Location of the centroid of the tip mass.

$$J_0 = \frac{2}{3} \rho_0 c \int_{A_0} (c^2 + 3z_2^2) dz_2 dz_3, \qquad (20)$$

where  $A_0$  denotes the cross-sectional area of the tip mass. To proceed further, one must assume something about the geometry of the beam and of the tip mass. For the sake of being specific, suppose that the tip mass and beam have circular-cross sections of radius a and b, respectively, where it will be hypothesized that a > b. Thus, it follows from equation (20) that

$$J_0 = \frac{\pi}{6} \rho_0 ca \left( 3a^2 + 4c^2 \right) . \tag{21}$$

In addition, for the beam of circular cross-section, it may be verified that

$$A = \pi b^2$$
,  $I = \frac{\pi}{4} b^4$ . (22)

Therefore, in view of equations (21), (22), and  $m = 2\rho_0\pi ca^2$ , one finds from equation (5)

$$\mu = \frac{2\rho_0 a^2 c}{\rho \ell b^2}, \qquad \gamma = \frac{\rho_0 c a^2 (3a^2 + 4c^2)}{60b^2 \ell^3}$$
 (23)

Clearly, for a beam of given dimensions and density, the values of  $\mu$  and  $\gamma$  can be changed by varying a, c, and  $\rho_0$  either together or independently. However, because the essentially new parameter here is c, let it be supposed that  $\rho_0$  and a are held fixed throughout a given sequence of computations while c is varied. Thus, the objective of the present numerical computations is to determine  $Q_{\rm cr}$  as a function of the ratio  $\xi = 2{\rm c}/\ell$ , which is the ratio of the length of the tip mass to the length of the beam. Hence, one has, in place of equation (23),

$$\alpha = \frac{\xi}{2}$$
,  $\mu = \frac{\rho_0}{\rho} (\frac{a}{b})^2 \xi$ ,  $\gamma = \frac{\mu}{12} (\xi^2 + 3s^2)$ , (24)

where  $s = a/\ell$ .

Suppose further that, again for the purpose of being specific,  $b/\ell = 1/20$  and  $a/\ell = 1/10$ , so that a/b = 2. Then equation (24) becomes

$$\alpha = \xi/2$$
,  $\mu = 4p\xi$ ,  $\gamma = \frac{1}{3}p\xi(\xi^2 + 3/100)$ , (25)

where  $p = \rho_0/\rho$ . Upon using equation (25), equation (18) is now solved numerically for the critical load  $Q_{cr}$  as a function of the tip mass length parameter  $\xi$  for p = 1/10, 1/2, 1, and 2. The numerical procedure consists of selecting a value of Q and computing the corresponding values of the frequencies  $\omega$  for the first two modes of vibration. The value of Q is successively increased from zero until the first and second frequencies coalesce at the critical value,  $Q_{cr}$ . As long as  $Q < Q_{cr}$ , these frequencies are real numbers, but for  $Q > Q_{cr}$  they are complex conjugate numbers. In the present case, loss of stability through divergence is not possible. The onset of flutter is signaled at  $Q = Q_{cr}$ . The results of these numerical computations are shown in Figure 3, where the variation of  $Q_{cr}/\pi^2$  has been plotted versus  $\xi$  over the range  $0 \le \xi \le 1$  for the four stated values of p.

This figure reveals that, for the values of p considered, the value of  $Q_{cr}$  decreases monotonically with increasing  $\xi$  on the interval  $0 \le \xi \le 1$ . Moreover, the value of  $Q_{cr}$  initially decreases rapidly from Beck's value of  $Q_{cr} = 2.031\pi^2$  as  $\xi$  increases from zero. Indeed, at least for sufficiently small values of  $\xi$ , the rate of decrease of  $Q_{cr}$  increases as the density ratio p increases, i.e., the initial slope of the tangent to the curve in the  $Q_{cr}\xi$ -plane becomes increasingly negative as the value of  $\xi$  is increased. As the value of  $\xi$  is further increased, the slope of the tangent to a given  $Q_{cr}\xi$ -curve eventually tends to increase over the range of  $\xi$  considered. Therefore, for a given, sufficiently small value of  $\xi$ , the critical load

decreases as p is increased, but for a given sufficiently large value of p the critical load increases as p is augmented.

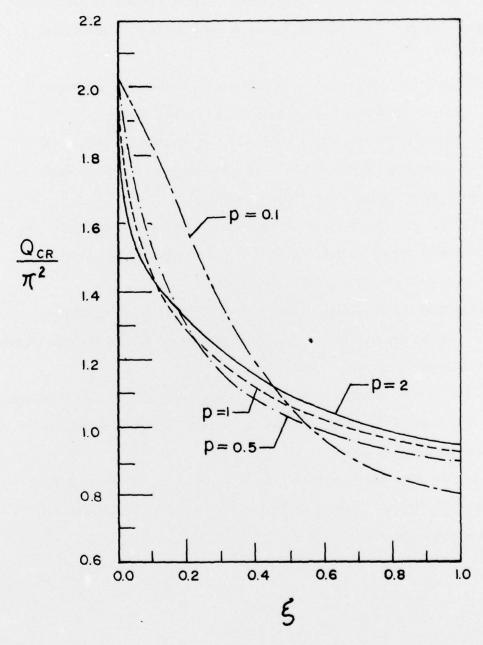


Figure 3. Variation of  $Q_{\rm cR}/\pi^2$  versus  $\xi$  for four values of p.  $_{11}$ 

#### REFERENCES

- M. Beck. 1952, Zeitschrift fur angewandte Mathematik und Physik 3, 225-228. Dir Knicklast des einseitig eingespannten tangential gedruckten Stabes. Errata: Zeitschrift fur angewandte Mathematik und Physik 3, 476-477.
- A. Pfluger, 1955, Zeitschrift fur angewandte Mathematik und Mechanik
   35, 191. Zur Stabilitat des tangential gedruckten Stabes.
- W. W. Walter and M. Levinson, 1968 Canadian Aeronautics and Space Institute Transactions 1, 91-93. Destabilization of a nonconservatively loaded elastic system due to rotary inertia.
- 4. G. L. Anderson, 1975 Journal of Sound and Vibration 43, 543-552. The influence of rotary inertia, tip mass, and damping on the stability of a cantilever beam on an elastic foundation.
- B. Rama Bhat and H. Wagner, 1976 Journal of Sound and Vibration 45, 304-307. Natural frequencies of a uniform cantilever with a tip mass slender in the axial direction.
- Rama Bhat and M. Avinash Kulkarni, 1976 American Institute of Aeronautics and Astronautics Journal 14, 536-537. Natural frequencies of a cantilever with a slender tip mass.
- A. H. Flax, 1978 American Institute of Aeronautics and Astronautics Journal 16, 94-96. Comment on "natural frequencies of a cantilever with slender tip mass".

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